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PENETRATION OF AN ELASTIC CIRCULAR CYLINDRICAL SHELL INTO AN INCOMPRESSIBLE LIQUID

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The plane unsteady problem of impact of a thin elastic cylindrical shell on the surface of an ideal incompressible liquid is considered. The initial stage of interaction between the body and the liquid when the stresses in the shell attain peak values is studied. The problem is treated in a linearized formulation and is solved numerically by the normal modes method within the framework of the Wagner approach. The numerical results agree with experimental data for various types of circular cylindrical shells made from mild steel.

The plane unsteady problem of an impact by an elastic circular cylindrical shell on the surface of an ideal incompressible liquid is considered. At the initial moment ($t = 0$), the liquid occupies the half-plane $y < 0$ and rests, and the shell touches the liquid at the point $x = 0$, $y = 0$ and has the velocity V directed vertically downward. The initial position of the free boundary of the liquid coincides with the horizontal line $y = 0$; the pressure on the free surface is assumed to be constant and equal to zero. For $t > 0$, the shell strikes the liquid. The shell model that takes into account the circumferential bending and normal stresses is used. The shell thickness h is assumed to be much smaller than its radius R. It is required to determine the stresses in the shell during its immersion in the liquid.

In this paper, we use the method of [1], which was developed for analysis of a wave impact on the center of a plate. However, the geometry of the problem is more complicated; therefore, it is necessary to use the cylindrical coordinates to describe the dynamics of a shell and the Cartesian coordinates to describe a liquid flow. Moreover, in contrast to the problem of a plate impact, where the duration of the impact stage can be estimated from geometrical considerations, in the case of a shell impact the impact stage is not distinct. It is assumed that the duration of the impact stage, in which the hydrodynamic loads are very high and the bending stresses reach their maximum value, is connected with the frequencies of free oscillations of the shell.

Introduction. We consider a thin circular cylindrical shell. Its strain state is plane if the deflection of the shell and the stresses in it do not vary along the generatrix of the shell. In deriving the equations of equilibrium of a plane circular cylindrical shell, the inertia in the circumferential and radial directions is taken into account (r and θ are the cylindrical coordinates). The dynamic equations for a circular shell are written in the form [2]

$$\frac{1}{R}\frac{\partial M}{\partial \theta} - \frac{\partial N}{\partial \theta} - \rho_0 h R \frac{\partial^2 v}{\partial t^2} = 0, \quad \frac{1}{R}\frac{\partial^2 M}{\partial \theta^2} + N + \rho_0 h R \frac{\partial^2 w}{\partial t^2} - p_0 R = 0.$$

Here w is the normal displacement of the shell, which is positive if directed toward the center, v is the tangential displacement, which is positive in the θ-direction, ρ_0 is the density of the shell material, p_0 is the external pressure acting on the shell, M is the specific circumferential bending moment, and N is the specific circumferential normal force.

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The specific forces in the shell cross sections are expressed in terms of the curvature change æ and the circumferential strain ε of the middle surface:

$$M = -\frac{Eh^3}{12(1-\nu^2)}æ, \qquad N = -\frac{Eh}{1-\nu^2}\varepsilon. \tag{1}$$

Here

$$æ = -\frac{1}{R^2}\Big(\frac{\partial v}{\partial \theta} + \frac{\partial^2 w}{\partial \theta^2}\Big); \qquad \varepsilon = \frac{1}{R}\Big(\frac{\partial v}{\partial \theta} - w\Big); \tag{2}$$

E is Young's modulus, and ν is Poisson's ratio of the shell material.

It should be noted that since the thickness of the shell is small, even insignificant bending moments cause high stresses; therefore, it is important to determine the forces in the shell upon bending to design a reliable structure.

The strain at an arbitrary point located at the distance z from the middle line of a plane shell is expressed in terms of the curvature change æ and the strain ε of the middle surface by the formula [2] $\varepsilon^z = \varepsilon + zæ$ $(-h/2 \leqslant z \leqslant h/2)$, where the z axis is directed toward the center of curvature of the shell.

In a dynamic analysis of shells, the hypothesis on the inextensibility of the middle line is widely used; this hypothesis implies that the circumferential strain of the middle line of a cylindrical shell ε is equal to zero and, hence, the equality $w = \partial v/\partial\theta$ holds. This assumption substantially simplifies the dynamic equations of a shell, but leads to certain inaccuracies in the solution of the problem, which will be considered below.

Under the condition that the shell's middle line is inextensible, the frequencies of its free oscillations are calculated from the formula [3]

$$f_i = \frac{1}{2\pi}\sqrt{\frac{E}{\rho_0(1-\nu^2)}\,\frac{h^2}{12R^4}\,\frac{i^2(i^2-1)^2}{i^2+1}}. \tag{3}$$

If $i = 0$ (radial oscillations), we obtain $f_0 = 0$; if $i = 1$, $f_1 = 0$ and the shell behaves like a rigid nondeformable body; if $i = 2$, the shell executes flexural oscillations corresponding to the main (lowest) normal mode with the period of oscillations $T_2 = 1/f_2$.

The hydrodynamic loads on impact of the shell on the liquid surface increase rapidly and then decay. Estimating the duration of the impact stage, we note that it can be determined as a stage of penetration of a body in a liquid when the dimension of the wetted part of the shell $S(t)$ is much larger than the depth of its immersion $H(t)$, i.e., the ratio $\sigma = H(t)/S(t)$ is small. It is noteworthy that this ratio depends on the solution of the problem and its smallness should be verified *a posteriori*. For a constant velocity of the body, we have $H(t) = Vt$. Neglecting the deformations of the liquid boundary and elastic shell during impact, we find that at the time t_1, the width of the contact region between the liquid and the body $S(t_1)$ is equal to $2\sqrt{R^2 - (R - Vt_1)^2}$ and is of the order $O(\sqrt{RVt_1})$ for small t_1. The quantity $L = \sqrt{RVT}$ is assumed to be the scale of length, where T is the scale of time, which is determined by the duration of the impact stage. If the quantity R/V is chosen as the scale of time, we obtain a crude estimate of the duration of the initial stage of penetration. It follows from (1) and (2) that within the framework of the hypotheses on the inextensibility of the middle line of a shell, the second mode of oscillations of the shell gives the main contribution to bending stresses. Therefore, the period of the lowest mode of flexural oscillations of the shell T_2 is taken to be the scale of time $T = T_2$.

The distinguishing feature of the problem is that the elastic shell is bent by the hydrodynamic loads the region of application of which $-c(t) < x < c(t)$ extends with time and whose amplitude depends on elastic strains. The problem is coupled: the liquid flow and the deformations of the body should be determined simultaneously. At the same time, it is necessary to determine the dimension of the wetted part of the body, which is an important characteristic of the process. The calculation of the function $c(t)$ involves significant difficulties and is usually carried out using approximations [4].

It is required to determine the strains of the shell, its deflections, and the position of the points of contact under the following assumptions:

1) The liquid is ideal and incompressible;

2) The external mass forces and the forces of surface tension are absent;

3) The liquid flow is planar, potential, and symmetric about the y axis;

4) The shell is elastic, thin-walled, and the deformations during the impact are small compared to the radius of the shell R;

5) The period of the lowest mode of free flexural oscillations of the shell T_2 is much smaller than the ratio R/V;

6) The dimension of the contact region between the shell and the liquid increases monotonically.

Below, we use the dimensionless variables that are determined using the following scales: L is the scale of length, T is the scale of time, V is the scale of velocity, VL is the scale of velocity potential, VT is the scale of displacements, and $\rho VL/T$ is the scale of pressure, where ρ is the density of the liquid. The dimensionless variables are denoted by the same symbols as the corresponding dimensional quantities.

Formulation of the Problem. In considering the initial stage of immersion, we assume that the vertical velocity of liquid particles is of the order of the velocity of the shell V. Then, the rise of the free boundary is of the order of the immersion depth of the shell $O(H)$. In this case, the linearization parameter σ (the ratio of the characteristic depth of immersion VT to the characteristic dimension of the contact region L in the impact stage) is equal to $\sqrt{VT/R}$. In the initial stage, when the depth of immersion is still small, the equation of motion and the boundary conditions can be linearized with accuracy $O(\sigma)$ and shifted to the initial level of the undisturbed liquid $y = 0$. The error of the approximate solution of this problem relative to the exact solution can be estimated as $O(\sigma)$. In the first approximation, the motion of the liquid is described by the Laplace equation for the velocity potential $\varphi(x, y, t)$ in the lower half-plane $y < 0$ with the corresponding boundary conditions.

In the symmetric case, the position of the points of contact is specified by one function $c(t)$. Although the equations of motion and the boundary conditions are linearized, the problem remains nonlinear, since the quantity $c(t)$ is not known beforehand. Precisely the last circumstance determines the difficulties that arise in a study of the impact of elastic bodies on the liquid.

In dimensionless variables, the formulation of the problem has the form

$$\frac{\partial^2 w}{\partial t^2} + A\Big(w - \frac{\partial v}{\partial \theta}\Big) + B\Big(\frac{\partial^3 v}{\partial \theta^3} + \frac{\partial^4 w}{\partial \theta^4}\Big) = G p_0(\theta, t) \quad (-\pi < \theta < \pi); \tag{4}$$

$$\frac{\partial^2 v}{\partial t^2} + A\Big(\frac{\partial w}{\partial \theta} - \frac{\partial^2 v}{\partial \theta^2}\Big) - B\Big(\frac{\partial^2 v}{\partial \theta^2} + \frac{\partial^3 w}{\partial \theta^3}\Big) = 0 \quad (-\pi < \theta < \pi); \tag{5}$$

$$v(\theta, 0) = v_t(\theta, 0) = w(\theta, 0) = w_t(\theta, 0) = 0 \qquad (-\pi < \theta < \pi); \tag{6}$$

$$p = -\varphi_t \qquad (y \leqslant 0); \tag{7}$$

$$\varphi_{xx} + \varphi_{yy} = 0 \qquad (y < 0); \tag{8}$$

$$\varphi = 0 \qquad (y = 0,\ \ |x| > c(t)); \tag{9}$$

$$\varphi_y = -1 + w_t \qquad (y = 0,\ \ |x| < c(t)); \tag{10}$$

$$\varphi \to 0 \qquad (x^2 + y^2 \to \infty). \tag{11}$$

Here $p(x, y, t)$ is the pressure in the liquid and $p_0(\theta, t)$ is the hydrodynamic load (pressure) acting on the shell. In the contact region, for $|x| < c(t)$ we have $p_0(\theta, t) = p(x(\theta, t), y(\theta, t), t)$, where $x(\theta, t)$ and $y(\theta, t)$ are the horizontal and vertical components of the moving deformable shell in the Cartesian coordinate system, respectively. Formula (7) for the pressure follows from the linearized Cauchy–Lagrange integral.

The dimensionless parameters A, B, and G in Eqs. (4) and (5) have the form

$$A = \frac{ET^2}{\rho_0 R^2(1-\nu^2)}, \qquad B = \frac{ET^2h^2}{12\rho_0 R^4(1-\nu^2)}, \qquad G = \frac{\rho L}{\rho_0 h}. \tag{12}$$

In the initial stage, we have $y(\theta, t) = O(\sigma)$ in the contact region and, hence,

$$p_0(\theta, t) \approx p(x(\theta), 0, t) \qquad [|\theta| < \theta_c(t)]. \tag{13}$$

Moreover, in this region, the following approximation in the principal order

$$\theta \approx \gamma x, \qquad \theta_c(t) \approx \gamma c(t), \qquad \gamma = L/R \qquad [t > 0, \ |x| < c(t)], \tag{14}$$

is valid as $\sigma \to 0$, which will be used below in the replacement of θ by x in the equations.

The formulation of problem (4)–(11) is not complete. Therefore, a condition that serves to determine the function $c(t)$ and calls for the nonpenetration of the liquid particles at the free surface into the moving elastic surface should be added. In the symmetric case, this condition leads to the transcendental equation [5]

$$\int_0^{\pi/2} y_b(c(t)\sin\theta, t)\, d\theta = 0, \tag{15}$$

which was derived using the Wagner condition [6]. Here the function $y_b(x, t)$ describes the shape of a moving deformable elastic shell. In dimensionless variables and in the principal order as $\sigma \to 0$, this function has the form $y_b(x, t) = x^2/2 - t + w(x, t)$. In this case, Eq. (15) yields

$$t = \frac{1}{4}c^2 + \frac{2}{\pi}\int_0^{\pi/2} w(c(t)\sin\theta, t)\, d\theta. \tag{16}$$

The initial boundary-value problem (4)–(16) is solved by the normal modes method.

Method of Solving the Problem. In the section of the liquid boundary $-\pi < x < \pi$, $y = 0$, which contains the spot of contact, the velocity potential and the pressure distribution can be represented in the form

$$\varphi(x(\theta), 0, t) = \sum_{n=0}^{\infty} \varphi_n(t)\cos(n\theta), \qquad p(x(\theta), 0, t) = \sum_{n=0}^{\infty} p_n(t)\cos(n\theta). \tag{17}$$

With allowance for (7), it follows that $p_n(t) = -\dot{\varphi}_n(t)$ $(n = 1, 2, \ldots)$. The dot denotes the derivative with respect to time.

The shell displacements are determined in the form

$$w(\theta, t) = \sum_{n=0}^{\infty} a_n(t)\cos(n\theta), \qquad v(\theta, t) = \sum_{n=1}^{\infty} b_n(t)\sin(n\theta). \tag{18}$$

Substituting expressions (17) and (18) into the equations of shell deformations (4) and (5), we obtain the following system of second-order ordinary differential equations:

$$\begin{gathered} \ddot{a}_n + a_n(A + Bn^4) - b_n(An + Bn^3) - Gp_n = 0, \\ \ddot{b}_n - a_n(An + Bn^3) + b_n(An^2 + Bn^2) = 0 \end{gathered} \tag{19}$$

for $n > 0$ and

$$\ddot{a}_0 + Aa_0 - Gp_0 = 0 \tag{20}$$

for $n = 0$.

It is convenient to use the generalized coordinates $a_n(t)$ $(n = 0, 1, 2, \ldots$ and $b_n(t)$ $(n = 1, 2, \ldots)$ as the new desired functions and express the other quantities in terms of these functions.

We write system (19), (20) in the form

$$\frac{d}{dt}\,(\dot{a}_n + G\varphi_n) + a_n(A + Bn^4) - b_n(An + Bn^3) = 0,$$
$$\frac{d}{dt}\,\dot{b}_n - a_n(An + Bn^3) + b_n(An^2 + Bn^2) = 0. \tag{21}$$

Now we can introduce new auxiliary functions $g_n = \dot{a}_n + G\varphi_n$ $(n = 0, 1, 2, \ldots)$ and $r_n = \dot{b}_n$ $(n = 1, 2, \ldots)$ and rewrite (21) in the form of a system of first-order ordinary differential equations

$$\dot{a}_n = g_n - G\varphi_n, \qquad \dot{g}_n = -a_n(A + Bn^4) + b_n(An + Bn^3),$$
$$\dot{b}_n = r_n, \qquad \dot{r}_n = a_n(An + Bn^3) - b_n(An^2 + Bn^2). \tag{22}$$

The eigenfunctions $\psi_n(\theta) = \cos(n\theta)$ satisfy the orthogonality condition

$$\frac{1}{\pi}\int\limits_{-\pi}^{\pi} \psi_n(\theta)\psi_m(\theta)\,d\theta = \delta_{nm},$$

where $\delta_{nm} = 0$ for $n \neq m$, $\delta_{nn} = 1$ for $n > 0$, and $\delta_{00} = 2$. With allowance for (9), (14), and (17), from this condition follow the equalities

$$\varphi_n(t) = \frac{\gamma}{\pi}\int\limits_{-c(t)}^{c(t)} \varphi(x, 0, t)\psi_n(\gamma x)\,dx \quad (n \neq 0), \qquad \varphi_0(t) = \frac{\gamma}{2\pi}\int\limits_{-c(t)}^{c(t)} \varphi(x, 0, t)\,dx.$$

To determine the dependences of $\varphi_m(t)$ $(m = 0, 1, 2, \ldots)$ on the generalized coordinates $a_n(t)$ $(n = 0, 1, 2, \ldots)$ and $b_n(t)$ $(n = 1, 2, \ldots)$, we consider the hydrodynamic part of the problem (4)–(11) separately.

We find new, harmonic in the lower half-plane $y \leqslant 0$, functions $\phi_n(x, y, c)$ as the solutions of the boundary-value problem

$$\frac{\partial^2\phi_n}{\partial x^2} + \frac{\partial^2\phi_n}{\partial y^2} = 0 \qquad (y < 0); \tag{23}$$

$$\phi_n = 0 \qquad [y = 0,\ \ |x| > c(t)]; \tag{24}$$

$$\frac{\partial\phi_n}{\partial y} = \psi_n(\gamma x) \qquad [y = 0,\ \ |x| < c(t)]; \tag{25}$$

$$\phi_n \to 0 \qquad (x^2 + y^2 \to \infty) \tag{26}$$

with integrable singularities of the first-order derivatives in the neighborhood of the points of change of the form of the boundary condition $(x = \pm c)$. Here $n = 0, 1, 2$, etc., and $\psi_0(\gamma x) \equiv 1$. One can note that given the function $c(t)$, problem (8)–(11) is linear. Comparing the boundary conditions (10) and (25), we obtain

$$\varphi(x, 0, t) = -\phi_0(x, 0, c) + \sum_{n=0}^{\infty} \dot{a}_n(t)\phi_n(x, 0, c); \tag{27}$$

$$\varphi_m(t) = -f_m(c) + \sum_{n=0}^{\infty} \dot{a}_n(t)S_{nm}(c). \tag{28}$$

Here

$$f_m(c) = \frac{\gamma}{\pi}\int\limits_{-c}^{c} \phi_0(x, 0, c)\psi_m(\gamma x)\,dx \qquad (m \neq 0); \tag{29}$$

$$f_0(c) = \frac{\gamma}{2\pi}\int\limits_{-c}^{c} \phi_0(x, 0, c)\,dx; \tag{30}$$

$$S_{nm}(c) = \frac{\gamma}{\pi} \int_{-c}^{c} \phi_n(x, 0, c) \psi_m(\gamma x)\, dx \qquad (m \neq 0, \quad n \neq 0); \tag{31}$$

$$S_{n0}(c) = \frac{\gamma}{2\pi} \int_{-c}^{c} \phi_n(x, 0, c)\, dx; \tag{32}$$

$$S_{0m}(c) = \frac{\gamma}{2\pi} \int_{-c}^{c} \phi_0(x, 0, c) \psi_m(\gamma x)\, dx. \tag{33}$$

The matrix S with the elements $S_{nm}(c)$ ($n = 0, 1, 2, \ldots$ and $m = 0, 1, 2, \ldots$) is symmetric, which follows from (25), (31)–(33), and the Green's second integral theorem, and depends only on the dimension of the spot of contact c. It is known that $\phi_0(x, 0, c) = \sqrt{c^2 - x^2}$ for $|x| < c$; whence $f_m(c) = cJ_1(m\gamma c)/m$ for $m \neq 0$ and $f_0(c) = c^2\gamma/4$, where J_1 is a first-order Bessel function. The elements S_{nm} of the matrix S can also be written in terms of the Bessel functions:

$$S_{nm}(c) = \frac{c}{n^2 - m^2} [nJ_1(nc\gamma)J_0(mc\gamma) - mJ_0(nc\gamma)J_1(mc\gamma)] \quad (n, m > 0, \quad n \neq m),$$

$$S_{n0}(c) = \frac{c}{2n} J_1(nc\gamma) \quad (n > 0), \qquad S_{nn} = \frac{c^2\gamma}{2} [J_0^2(nc\gamma) + J_1^2(nc\gamma)] \quad (n > 0), \qquad S_{00} = \frac{\gamma c^2}{4},$$

which significantly simplifies their calculation. Substituting (28) into system (22), we obtain the following infinite system of ordinary differential equations relative to the generalized coordinates:

$$\frac{d\boldsymbol{a}}{dt} = (I + GS)^{-1}(\boldsymbol{g} + G\boldsymbol{f}); \tag{34}$$

$$\frac{d\boldsymbol{g}}{dt} = -D_1\boldsymbol{a} + D_2\boldsymbol{b}; \tag{35}$$

$$\frac{d\boldsymbol{b}}{dt} = \boldsymbol{r}; \tag{36}$$

$$\frac{d\boldsymbol{r}}{dt} = D_2\boldsymbol{a} - D_3\boldsymbol{b}. \tag{37}$$

Here $\boldsymbol{a} = (a_0, a_1, a_2, \ldots)^{\mathrm{t}}$; $\boldsymbol{b} = (b_1, b_2, \ldots)^{\mathrm{t}}$, $\boldsymbol{g} = (g_0, g_1, g_2, \ldots)^{\mathrm{t}}$, $\boldsymbol{r} = (r_1, r_2, \ldots)^{\mathrm{t}}$, $\boldsymbol{f} = (f_0(c), f_1(c), f_2(c), \ldots)^{\mathrm{t}}$, I is a unit matrix, D_1, D_2 and D_3 are diagonal matrices, $D_1 = \mathrm{diag}\{A + Bn^4\}$, $D_2 = \mathrm{diag}\{An + Bn^3\}$, and $D_3 = \mathrm{diag}\{An^2 + Bn^2\}$, and the superscript t denotes transpose of a matrix.

The right sides in system (34)–(37) depend on $\boldsymbol{a}$, $\boldsymbol{b}$ $\boldsymbol{g}$, $\boldsymbol{r}$, and c and do not depend on t; therefore, it is convenient to choose the quantity c as a new independent variable. The differential equation for $t = t(c)$ follows from condition (19) if one differentiates it with respect to c:

$$\frac{dt}{dc} = Q(c, \boldsymbol{a}, \dot{\boldsymbol{a}}). \tag{38}$$

Here $Q(c, \boldsymbol{a}, \dot{\boldsymbol{a}}) = (c + (4/\pi)(\boldsymbol{a}, \boldsymbol{\Gamma}_c(c)))/(2 - (4/\pi)(\dot{\boldsymbol{a}}, \boldsymbol{\Gamma}(c)))$. Here $(\cdot, \cdot)$ is the scalar product of the vectors, $\boldsymbol{\Gamma}(c) = (\Gamma_0(c), \Gamma_1(c), \Gamma_2(c), \ldots)$, $\boldsymbol{\Gamma}_c(c) = (\Gamma_{0c}(c), \Gamma_{1c}(c), \Gamma_{2c}(c), \ldots)$, $\Gamma_n(c) = \int_0^{\pi/2} \psi_n(c \sin\theta)\, d\theta$, and $\Gamma_{nc}(c) = \int_0^{\pi/2} \psi_n'(c \sin\theta) \sin\theta\, d\theta$ which, after calculations, can be expressed in terms of zeroth- and first-order Bessel functions:

$$\Gamma_n(c) = \frac{\pi}{2} J_0(nc\gamma), \qquad \Gamma_{nc}(c) = -\frac{\pi}{2} n\gamma J_1(nc\gamma).$$

Multiplying each equation of system (34)–(37) by dt/dc and making allowance for (38), we obtain

$$\frac{d\boldsymbol{a}}{dc} = \boldsymbol{F}(c, \boldsymbol{g})Q(c, \boldsymbol{a}, \boldsymbol{F}(c, \boldsymbol{g})), \qquad \frac{d\boldsymbol{g}}{dc} = (-D_1\boldsymbol{a} + D_2\boldsymbol{b})Q(c, \boldsymbol{a}, \boldsymbol{F}(c, \boldsymbol{g})),$$
$$\frac{d\boldsymbol{b}}{dc} = \boldsymbol{r}Q(c, \boldsymbol{a}, \boldsymbol{F}(c, \boldsymbol{g})), \qquad \frac{d\boldsymbol{r}}{dc} = (D_2\boldsymbol{a} - D_3\boldsymbol{b})Q(c, \boldsymbol{a}, \boldsymbol{F}(c, \boldsymbol{g})). \tag{39}$$

Here $\boldsymbol{F}(c, \boldsymbol{g}) = (I + GS)^{-1}(\boldsymbol{g} + G\boldsymbol{f}(c))$. System (39) is solved numerically for the zero initial conditions

$$\boldsymbol{a} = 0, \qquad \boldsymbol{b} = 0, \qquad \boldsymbol{g} = 0, \qquad \boldsymbol{r} = 0, \qquad t = 0 \qquad (c = 0). \tag{40}$$

The choice of c as a new independent variable seems to be natural, since this corresponds to the structure of system (39). The introduction of new desired functions $g_n(t)$ and $r_n(t)$ instead of the derivatives $\dot{a}_n(t)$ and $\dot{b}_n(t)$ solves the problem of the beginning of numerical calculation: the right sides in system (38), (39) vanish for $c = 0$. If the problem is solved in the initial variables, difficulties with the beginning of calculation arise, which can be overcome only by an artificial method. The reason is that, for short times, we have $c(t) = O(\sqrt{t})$, $w(x, t) = O(t^{3/2})$, $w_t = O(\sqrt{t})$, and $w_{tt} = O(t^{-1/2})$, i.e., in the beginning of the impact, the spot of contact extends with a very large velocity, and the accelerations of the elements of the elastic surface are unbounded. At the same time, the desired quantities, which are regarded as functions of c, i.e., $t = O(c^2)$, $w = O(c^3)$, and $w_t = O(c)$, increase smoothly at the initial stage. In solving the Cauchy problem (39), (40), the derivatives $\dot{a}_n(t)$ are determined by formula (34), and the derivatives $\dot{b}_n$ by formula (36).

In the Cauchy problem (39), (40), a finite number of normal modes N is retained and it is assumed that $a_n \equiv 0$, $b_n \equiv 0$, $g_n \equiv 0$, and $r_n \equiv 0$ for $n \geqslant N + 1$. The problem is solved by the fourth-order Runge–Kutta method with a constant step relative to the variable c. The choice of the step was considered in detail in [1]. Calculations were performed for various values of N to investigate the convergence with increasing N. It is sufficient to take $N = 15$ in the calculations, since the values of the desired quantities vary insignificantly as the number of the retained modes is increased.

Description of the Experiment. Shibue et al. [7] carried out experiments on a thin-walled cylinder to reveal the specific features of the deformation of a cylinder at water impact. Faltinsen et al. [7] proposed a numerical method of determining the deformations of a cylinder with the use of the available experimental data. The pressure measured in the experiments is divided into two parts, each of which is used to estimate the maximum values of the strains. The pressure in the shell increases rapidly on water impact and then decreases abruptly during its further immersion. This part of the pressure with a pronounced peak is called an impact pressure. The profile of the impact pressure is approximated by a triangular pulse, the base and height of the triangle corresponding to the duration and amplitude of the impact pressure, respectively. After the impact pressure is decreased, the hydrodynamic loads acting on the body are relatively small, but act for a long time. The experiments of [7] show that the strains that were obtained numerically only with allowance for the impact pressure are significantly smaller than the measured values. Consequently, the peakless part of the pressure should also be taken into account in calculations.

The purpose of our experiment is to investigate the evolution of the hydrodynamic pressure on an elastic cylinder and its stresses in the two-dimensional case.

A cylinder of length 600 mm and diameter 312 mm was dropped on the water surface from heights of 0.5, 1.0, and 1.5 m. To provide a two-dimensional flow during immersion of the body in water, two vertical plates were installed on either side of the cylinder. To preserve the horizontal position of the cylinder during its fall, the cylinder was suspended from a rigid beam sliding along two vertical guides. The cylinder was lifted to the required height by means an electromagnet, then the magnet was switched off, and the cylinder fell down to the water. At the moment when the cylinder touched the water, it was detached from the beam and moved without the beam. To prevent the leakage of water into the cylinder, a rubber film was pulled on the ends of the cylinder.

The experiment was carried out on cylindrical shells of thicknesses 5.1 and 1.0 mm; we call them thick and thin cylinders, respectively. The cylinders were made of mild steel of different standards; the mass of the thick cylinder (JIS STRG370 standard) was 23.8 kg and the mass of the thin cylinder (JIS SPCC standard) was 5.0 kg.

Strain gauges of the semiconductor type and diaphragm pressure gauges were attached to the inner surfaces of the cylinders; the thin cylinder was equipped only with a strain gauge. The signals from these gauges were registered by a personal computer. The threshold frequency registered by the pressure gauges was 100 kHz and that registered by the strain gauges was 5 kHz.

The main results of the experiment are given for the top of the shell ($\theta = 0$), i.e., for the point of initial contact between the body and the water surface; here, the maximum pressures and strains are observed.

For the thick cylinder, for $\theta = 0$ the strains and the pressure reach their maximum values for about 4 msec and 0.05 msec, respectively, after the impact. The measured period of oscillations of the cylinder is about 8 msec, which agrees with the value 7.5 msec predicted from Eq. (3) for the period of two-dimensional flexural oscillations of the cylinder. For the thin cylinder, the maximum strain is observed much later, for about 22 msec after the impact.

Numerical Results. In a numerical solution of the problem (39), (40), the experimental data of [7] were used. The values of the main parameters are $R = 0.156$ m, $h_1 = 5.1$ mm, $h_2 = 1.0$ mm, $m_1 = 23.8$ kg, $m_2 = 5.0$ kg, $E = 206 \cdot 10^9$ Pa, and $\nu = 0.33$. (The subscripts 1 and 2 denote the quantities that refer to the thick and thin cylinders, respectively.) These values are used to calculate the other parameters of the problem, namely, the densities of the shell material $\rho_{01} = 8067$ kg/m^3 and $\rho_{02} = 8530$ kg/m^3 under the assumption that the distribution of the mass over the body is uniform and the fall velocity of the shell is $V = 4.1$ m/sec under the assumption that its center of mass falls from a height of 1 m.

For the thick cylinder, the coefficients in the equations of the shell are $A = 6.17 \cdot 10^4$, $B = 5.5$, and $G = 1.65$. The scale of length is 6.8 cm, the scale of time is 7.2 msec, and the scale of velocity potential is 0.28 m^2/sec, the scale of displacement is 2.9 cm, and the scale of pressure is 0.038 N/mm^2, and the linearization parameter is $\sigma = 0.43$.

For the thin cylinder, we find that $A = 1.61 \cdot 10^6$, $B = 5.5$, and $G = 18.14$. The scale of length is 15.5 cm, the scale of time is 38 msec, the scale of velocity potential is 0.63 m^2/sec, the scale of displacement is 15.4 cm, the scale of pressure is 0.016 N/mm^2, and the linearization parameter is $\sigma = 0.995$. One should be critical about the numerical results obtained for the thin cylinder, since the corresponding scales are too large for the description of the initial stage of penetration of the elastic body into the liquid.

We now investigate the water impact of the thick cylinder in greater detail. The strains at the top of the cylinder ($\theta = 0$) were calculated. The calculations were performed for various numbers of the retained modes N to establish the convergence of the numerical solution with increase in N. The resulting time dependence of the strain ε in the thick cylinder for $\theta = 0$ and $V = 4.1$ m/sec agrees qualitatively with the experimental data (curve 1 in Fig. 1 refers to numerical results, and curve 2 to the experiment). The time at which the maximum strains occur in the cylinder is the same, but their numerical values differ considerably. Apparently, this is caused by the wrong choice of the velocity of the cylinder. Indeed, it follows from the experiments on the impact of a horizontal plate on the wave crest [8] that after the impact, the velocity of the body decreases abruptly up to a certain value and remains almost constant for a while. For example, in the experiment of [8], the velocity was equal to 3.1 m/sec before the impact and 2.5 m/sec after the impact. According to [8], precisely this velocity value should be used in the calculation of the stresses in the plate when it interacts with a liquid.

It is not clear from [7], the data of which were used in our study, whether the velocities of the cylinder during its fall and subsequent penetration into the liquid were measured. In this connection, we carried out further investigation of our model, namely, we calculated the maximum strain as a function of the impact velocity V for $\theta = 0$ (curve 1 in Fig. 2 refers to the quadratic approximation $y = 14.386x^2 + 45.139x$ with the correlation coefficient 0.998, curve 2 to the approximation $y = 141.58x - 121.31$ with the correlation

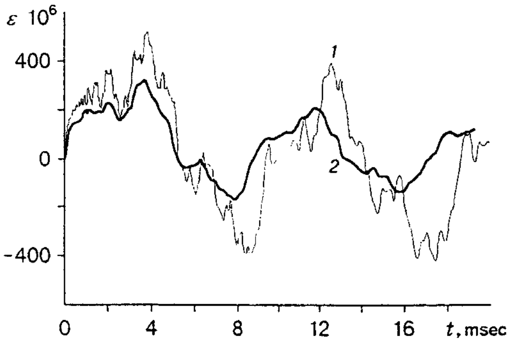

Fig. 1

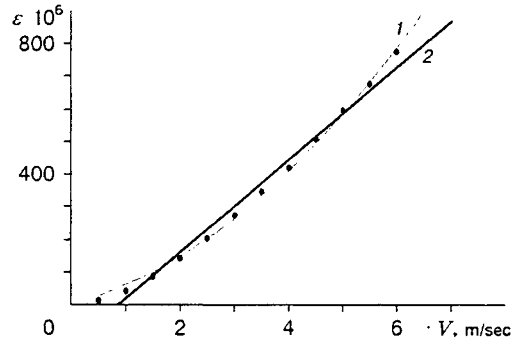

Fig. 2

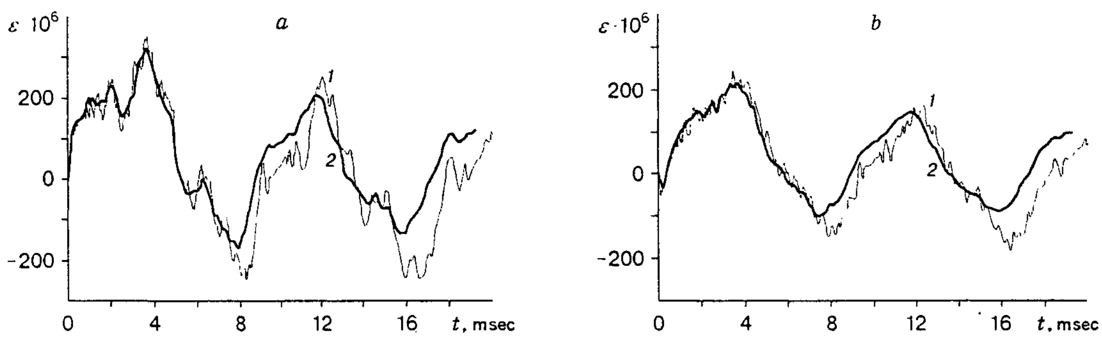

Fig. 3

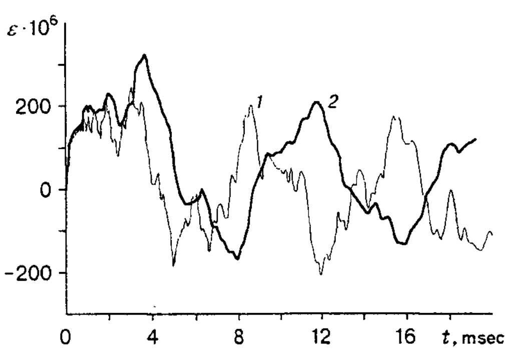

Fig. 4

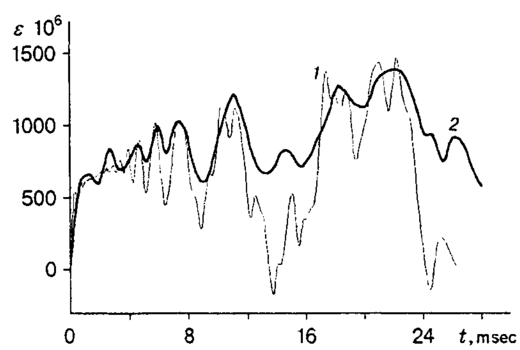

Fig. 5

coefficient 0.992, and the points refer to numerical calculation). In the experiment of [7], the maximum value of the strain was $\varepsilon = 322 \cdot 10^{-6}$ for $\theta = 0$; using this value, we found that $V = 3.5$ m/sec, which corresponds to the velocity of penetration of the cylinder into a liquid in our model. Further calculations for the thick cylinder were performed for $V = 3.5$m/sec.

For $V = 3.5$ m/sec, the scale of length is 6.3 cm, the scale of velocity potential is 0.22 m^2/sec, the scale of displacements is 2.5 cm, the scale of pressure is 0.03 $\mathrm{N/mm}^2$, and the linearization parameter is $\sigma = 0.40$.

The results of calculations for $V = 3.5$ m/sec shown in Fig. 3a and 3b for $\theta = 0$ and 20°, respectively (curves 1), agree well with the experimental data of [7] (curves 2) for the first 7–8 8 msec after the impact. Thus, the estimate of the maximum strains can be considered reliable.

Figure 4 shows results of the calculations for $V = 3.5$ m/sec that were obtained with the use of the approximate shell model (curve 1), i.e., under the assumption that the middle line of the shell is inextensible. The model gives a result close to the experiment (curve 2) only for the first 2.5–3 msec after the impact and yields incorrect values of the maximum strains. The latter is a serious drawback of the model.

Similar calculations were performed for the thin cylinder with $V = 4.1$ m/sec. The numerical results shown in Fig. 5 (curve 1) do not quite agree with the experimental results (curve 2), but give insight into the variation of the initial parameters of the problem.

Thus, the model of hydroelastic interaction between a cylindrical shell and a liquid that is used in the present study describes the process with sufficient accuracy and allows one to estimate the maximum strains of the shell. The number of modes which are necessary to retain to describe the dynamics of the shell is relatively small.

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